Performance Evaluation and Optimization of Traffic-Aware Cellular Multiuser Two-Way Relay Networks Over Nakagami-$m$ Fading

Mahendra K. Shukla, Student Member, IEEE, Suneel Yadav, Member, IEEE, and Neetesh Purohit, Member, IEEE

Abstract—We consider a cellular multiuser two-way relay network, where a multiantenna base station communicates with one of the several single-antenna mobile stations via a single-antenna relay terminal. We derive the exact and asymptotic overall outage probability (OOP) expressions of the considered system with asymmetric traffic under Nakagami-$m$ fading. Based on the asymptotic analysis, we formulate three optimization problems, namely, optimal power allocation (OPA) with fixed relay location, optimal relay location with fixed power allocation, and jointly OPA and relay location to minimize the OOP. Specifically, we investigate two OPA schemes, called OPA I and OPA II. To this end, OPA I provides the solutions for individual transmit powers at each terminal, whereas under OPA II, relay terminal is assigned with half of the total power and the remaining power is allocated to the end terminals. The numerical and simulation results corroborate our theoretical findings, and reveal that under highly asymmetric traffic, OPA I can achieve a significant OOP performance than OPA II for different relay locations, except at the relay location where both coincide. Under symmetric traffic, OPA I outperforms OPA II, irrespective of relay locations. The OOP performance can be further improved if the power allocation and relay location are jointly optimized, especially under unbalanced conditions.

Index Terms—Asymmetric two-way relaying (TWR), Nakagami-$m$ fading, overall outage probability (OOP), power allocation, relay location.

I. INTRODUCTION

DESIGNING future wireless cellular networks by exploiting the advantages offered by the cooperative relaying techniques in terms of coverage, throughput, and reliability has recently gained considerable attention in the research community. In particular, two-way relaying (TWR) [1] has emerged as a spectral efficient scheme for half-duplex relaying networks. The simplest protocol for TWR is termed as analog network coding (ANC) [2]–[6], where two users can exchange information bidirectionally with two-phase (2P) transmissions via an amplify-and-forward (AF) based half-duplex relay. Since the ANC protocol allows simultaneous uplink and downlink transmissions, it can be adopted in future cellular networks to facilitate multiuser transmission. Of particular interest are the cellular multiuser TWR networks, where a base station (BS) communicates bidirectionally with one of the several mobile stations (MSs) via a relay terminal. For the traditional one-way cellular multiuser networks, various user scheduling schemes to exploit the multiuser diversity and to improve the system performance have been studied in several publications (see [7]–[9] and the references therein). Recently, considerable attention has been devoted to analyze the performance of the ANC-based cellular multiuser TWR networks [10]–[12]. Specifically, the performance of such networks with user scheduling has been studied in [10]. In [11], Li et al. have analyzed the performance of cellular TWR networks by employing multiple antennas at the BS with multiple relays. In [12], Nhat et al. have analyzed the performance of a cellular multiuser TWR network by assuming the multiple antennas at all terminals with multiuser scheduling.

However, most of the aforementioned studies have considered the symmetric traffic requirements. In practice, the data rate requirements may be different at the two end terminals for various services in most networks, e.g., web browsing has large traffic volume in the downlink direction. Therefore, it is worth to consider the impact of asymmetry traffic requirements in practical system designs. For instance, the outage performance of traffic-aware TWR networks has been investigated for Rayleigh fading in [13] and [14], and for Rician fading in [15]. For such networks, the outage performance under Rayleigh fading with fixed gain relaying has been analyzed in [16]. While most of the above studies have considered a basic TWR scenario with traffic asymmetry, little attention has been devoted to taking cellular multiuser scenario for TWR networks into consideration [17], [18]. In particular, the authors in [17] have analyzed the performance of a cellular multiuser TWR network with different rate requirements under Rayleigh fading. Budhiraja and Ramamurthi [18] have proposed a novel linear precoder at the AF relay and analyzed the performance of such networks with traffic asymmetry under Rayleigh fading. However, to the best of the authors’ knowledge, no effort has been directed to analyze the performance of the cellular multiuser TWR networks under generalized Nakagami-$m$ fading. It is well known that the Nakagami-$m$ fading has shown to be a good fit based on experimental observations and it covers a wide range of fading scenarios, which includes the Rayleigh fading ($m = 1$) as a special case. Therefore, it is of great significance to analyze the performance of the cellular multiuser asymmetric TWR networks under Nakagami-$m$ fading channels.

Furthermore, the power allocation and relay location are two crucial design aspects for relaying protocols to enhance the system performance, especially in view of the asymmetric traffic conditions. Even one of the interesting design objectives for such protocols is the joint optimization of power allocation and relay location. Such optimization problems for the TWR networks under symmetric traffic patterns have been investigated in the presence of Nakagami-$m$ fading [19], [20]. For traffic-aware TWR networks, separate optimization of relay location and power allocation have been studied under Rayleigh fading in [13], whereas the authors in [14] and [21]–[23] have...
investigated only the problem of power allocation optimization under Rayleigh fading. Recently, the authors in [17] have only considered the relay location optimization problem for the traffic-aware cellular multiuser TWR networks under Rayleigh fading. However, the optimization problem of power allocation and joint optimization of power allocation and relay location for the cellular multiuser asymmetric TWR networks are still an open issue, even for the popular case of Rayleigh fading. Our aim is to fill this important gap.

With above motivations, we concentrate on a cellular multiuser TWR network where one multiantenna BS communicates bidirectionally with one of the several single-antenna MSs via one single-antenna relay. In particular, we investigate the impact of traffic asymmetry on the overall outage performance of the considered system under Nakagami-\(m\) fading. First, we formulate the instantaneous end-to-end signal-to-noise ratios (SNRs) by employing beamforming with maximum ratio transmission (MRT) and maximum ratio combining (MRC) at the BS and user scheduling (MSs). Then, we evaluate the overall outage probability (OOP) and the asymptotic outage behavior under traffic asymmetry. In addition, we deal with the separate and joint optimization of power allocation and relay location problems to enhance the OOP performance. The contributions of this paper can be summarized as follows.

1) For the considered system, we derive the OOP expression based on the extent of traffic asymmetry under Nakagami-\(m\) fading. Such expression can only be determined by the one-way channel or two-way transmissions, depending upon the different level of the two end terminals’ target rates and terminals’ average transmission powers.

2) We also conduct an asymptotic analysis for the OOP in the high SNR regime to highlight the impact of key system/channel parameters on the system diversity performance. By doing so, we show that the achievable diversity order is \(\min(m_a, N_a, K)\), where \(m_a\) and \(m_b\) represent the fading severity parameters between \(N_a\)-antenna BS and relay and between the \(K\) single-antenna MSs and relay, respectively. Furthermore, we show that the achievable diversity order is independent of traffic asymmetry.

3) Moreover, we present two optimal power allocation (OPA) schemes under fixed relay location (i.e., OPA I and OPA II) to minimize the OOP. To this end, OPA I studies the power allocation at each terminal, whereas OPA II studies the power allocation at the two end terminals with fixed half of the total power at the relay terminal. We also investigate the problem of optimal relay location under fixed power allocation to minimize the OOP. Then, under these power allocation schemes, we formulate two joint optimization problems of power allocation and relay location to enhance the OOP performance.

The remainder of this paper is organized as follows. In Section II, we describe our cellular multiuser TWR network setup. In Section III, we derive the expressions of the OOP and asymptotic outage probability for the considered system under Nakagami-\(m\) fading. The optimization of power allocation and relay location are investigated in Section IV. Section V presents the numerical and simulation results, and finally, the conclusions are drawn in Section VI.

Notations: Let \(Z \sim \text{Nak}(m, \Omega)\) denotes a Nakagami random variable (RV), and \(W \sim |Z|^2\) is a Gamma RV with fading severity parameter \(m\) and average power \(\Omega\). In general, the probability density function (PDF) and cumulative distribution function (CDF) of \(W\), are given, respectively, by

\[
f_W(w) = \frac{1}{\Gamma(m)} w^{m-1} e^{-w/m}, \quad \text{and} \quad F_W(w) = \frac{1}{\Gamma(m)} \Gamma(m, w/m).
\]

Table I lists the parameters/symbols used in this paper.

<table>
<thead>
<tr>
<th>Parameters/Symbols</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>(P_a, P_b, \text{and } P_t)</td>
<td>Transmit powers at terminals (T_a, T_b), and (R)</td>
</tr>
<tr>
<td>(N_0)</td>
<td>AWGN variance at each link</td>
</tr>
<tr>
<td>(N_a)</td>
<td>Number of antennas at the BS</td>
</tr>
<tr>
<td>(K)</td>
<td>Number of MSs</td>
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<tr>
<td>(m_a) and (m_b)</td>
<td>Fading severity parameters</td>
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<tr>
<td>(\alpha)</td>
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<td>(1/\alpha) and (\alpha)</td>
<td>Transpose and conjugate transpose</td>
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<tr>
<td>(\Omega)</td>
<td>(N \times N) identity matrix</td>
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<td>(</td>
<td></td>
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<tr>
<td>(T(\cdot)) and (F(\cdot))</td>
<td>Lower incomplete and complete gamma functions [24, eq. (8.380)]</td>
</tr>
<tr>
<td>(K_0())</td>
<td>(K)-th order modified Bessel function of second kind [24, eq. (8.432.6)]</td>
</tr>
<tr>
<td>(E_n())</td>
<td>Exponential integral function of order (n) [25, eq. (5.1.4)]</td>
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</tbody>
</table>

![System model of the cellular multiuser TWR](image)

**Fig. 1.** System model of the cellular multiuser TWR.

\[(\frac{m}{\pi})^m w^{-\frac{1}{2}} e^{-\frac{w}{m}}\] and \(F_W(w) = \frac{1}{\Gamma(m)} \Gamma(m, w/m)\). Table I lists the parameters/symbols used in this paper.

### II. System and Channel Models

We consider a cellular multiuser TWR network as shown in Fig. 1, consisting of an \(N_a\)-antenna BS \(T_a\), a single-antenna relay \(R\), and \(K\) single-antenna MSs \(T_{bk}\), \(k = 1, 2, \ldots, K\). Such network guarantees both multiantenna diversity and multiuser diversity. All terminals are assumed to operate in a half-duplex manner. The direct links between \(T_a\) and \(T_{bk}\) are absent due to large separation and heavy path loss. We assume that the channel for all links is quasi-static, reciprocal, and subject to independent and identically distributed (i.i.d.) Nakagami-\(m\) flat fading. Specifically, the channel vector for the link \(T_a - R\) is denoted by \(h_a = [h_a^1, h_a^2, \ldots, h_a^{N_a}]^t\), whose entries \(h_a^l\), \(l \in \{1, \ldots, N_a\}\) are modeled as i.i.d. \(\text{Nak}(m_a, \Omega_a)\) flat fading. Similarly, the channel vector for the link \(R - T_{bk}\) is denoted by \(h_{bk} = [h_{bk}^1, \ldots, h_{bk}^{N_a}]^t\), whose entries \(h_{bk}^l\), \(l \in \{1, \ldots, K\}\), which follow i.i.d. \(\text{Nak}(m_b, \Omega_b)\). We further assume additive white Gaussian noise (AWGN) with zero mean and variance \(N_0\) for each link.

The bidirectional communication between \(T_a\) and \(T_{bk}\) can be accomplished in two time phases by employing ANC-based TWR protocol with MRT/MRC beamforming at \(T_a\) and opportunistic user selection (\(T_{bk}\)). The user selection strategy will be discussed later in this section. In the first phase, the terminals \(T_a\) and \(T_{bk}\) simultaneously transmit their messages (denoted by \(x_a\) and \(x_{bk}\) with unit energies, respectively) with power \(P_a\) and \(P_b\), respectively, to \(R\). Hence, the received signal at \(R\) in the first phase is given by \(y_{r,k} = \sqrt{\beta/\Omega} |h_{bk}|^2 w_a x_a + \sqrt{\beta/\Omega} |h_a|^2 x_b + n_{r,k}\), where \(w_a = (\bar{x_a}/||h_a||)\) is the \(N_a \times 1\) transmit weight vector.
for MRT at \( T_n \) and \( n_{r,k} \) is AWGN at \( R_t \). During the second phase, the relay broadcasts the combined signal \( y_{r,k} \) after amplifying with the variable gain \( G_k \approx \sqrt{\frac{P_d}{\| h_k \|^2 + P_b h_b^k}} \) (adopted by ignoring the noise statistic at the relay [17]). \( P_d \) is the transmit power at the relay. The \( N_0 \times 1 \) received vector signal at \( T_n \) is multiplied by received weight vector \( w^k = (h_b^k / \| h_b^k \|) \) for MRC. Then, after self-interference cancelation the instantaneous end-to-end SNRs at \( T_{b_k} \) and \( T_a \) are expressed, respectively, as
\[
\Lambda_{ab_k} = \frac{P_a P_r}{N_0} \left[ \frac{\| h_b^k \|^2 h_b^k}{(P_a + P_r) \| h_b^k \|^2} \right] \tag{1}
\]
\[
\Lambda_{h_k} = \frac{P_a P_r}{N_0} \left[ \frac{\| h_b^k \|^2 h_b^k}{(P_a + P_r) \| h_b^k \|^2} \right]. \tag{2}
\]

For the considered system, an opportunistic user selection strategy is adopted to maximize the end-to-end SNRs \( \Lambda_{ab_k} \) and \( \Lambda_{h_k} \), and hence the overall system performance. Therefore, the best user \( (k^*) \) among all the active users is chosen based on the following criteria:
\[
k^* = \arg \max_k \{ \| h_b^k \|^2 \}.
\]

Therefore, with best user selection, we have \( \Lambda_{ab_k} = \frac{P_a P_r}{N_0} \Lambda_{ab_k} \) and \( \Lambda_{h_k} = \frac{P_a P_r}{N_0} \Lambda_{h_k} \), where \( \Lambda_{ab_k} = \frac{P_a}{\| h_b^k \|^2} \) and \( \Lambda_{h_k} = \frac{P_a}{\| h_b^k \|^2} \). The corresponding data rates \( T_a \rightarrow R \rightarrow T_{b_k} \) and \( T_a \leftarrow R \rightarrow T_{b_k} \) are depicted as \( R_{ab_k} = \frac{1}{2} \log_2 (1 + \Lambda_{ab_k}) \) and \( R_{h_k} = \frac{1}{2} \log_2 (1 + \Lambda_{h_k}) \), respectively, where the factor \( \frac{1}{2} \) is induced by the fact that the information exchange takes place in two transmission phases.

Based on the properties of beamforming and user selection, the CDFs and PDFs of \( \Lambda_{ab_k} \) and \( \Lambda_{h_k} \) are given as
\[
F_X (x) = \frac{1}{(m_a N_a x, m_m x)} \left( \frac{m_a N_a - m_m x}{X} \right) \tag{3}
\]
\[
F_Y (y) = \frac{1}{(m_m y, m_m y)} \left( \frac{m_m y, m_m y}{Y} \right)^K \tag{4}
\]
\[
f_X (x) = \frac{m_a N_a - m_m x}{m_a N_a x} \exp \left( \frac{m_m x}{X} \right) \tag{5}
\]
\[
f_Y (y) = \frac{K}{(m_m y)} \frac{m_m - 1}{(K - 1)} \sum_{p=0}^{k-1} \left( \begin{array}{c} K - 1 \end{array} \right) (-1)^p \times \sum_{r=0}^{p(m_m - 1)} b_p^r y^{m_m + r} e^{- \frac{m_m x}{y}} \tag{6}
\]
where \( (X, X) \in \{(\Lambda_{ab_1}, \bar{\Lambda}_{ab_1}), (\Lambda_{ab_2}, \bar{\Lambda}_{ab_2})\} \), \( (Y, Y) \in \{(\Lambda_{h_1}, \bar{\Lambda}_{h_1}), (\Lambda_{h_2}, \bar{\Lambda}_{h_2})\} \), and \( \bar{\Lambda}_{ab_1} = \frac{P_a + P_f}{N_0} \Omega_{ab_1}, \bar{\Lambda}_{ab_2} = \frac{P_a + P_f}{N_0} \Omega_{ab_2} \), \( \bar{\Lambda}_{h_1} = \frac{P_a}{N_0} \Omega_{h_1}, \) and \( \bar{\Lambda}_{h_2} = \frac{P_a}{N_0} \Omega_{h_2} \), and \( b_p^r = \sum_{i=0}^{r} \sum_{j=0}^{r-i} ((i + r + 1) a b_{p, i}^{r-i}) \).

III. ANALYTICAL STUDY OF OUTAGE PROBABILITY

Here, we derive closed-form expression for the OOP of the considered traffic-aware cellular multiuser TWR network under Nakagami-\( m \) fading. In order to assess the impact of traffic asymmetry, we derive the asymptotic outage probability in the high SNR regime.

A. Overall Outage Probability

The OOP is an important quality-of-service measure as it jointly characterizes the two outage events at the two end terminals. In fact, an overall outage event is said to occur when either \( R_{ab_k} \) or \( R_{h_k} \) is smaller than its target rate \( R_{th_k} \). According to (7), the exact closed-form expression of \( \alpha \) is obtained, which can be expressed in the following theorem.

**Theorem 1:** Under Nakagami-\( m \) fading, the closed-form expression for the OOP of the cellular multiuser TWR system with traffic asymmetry is given by
\[
\alpha = \begin{cases} \frac{P_{out,1}}{\Psi_{out,1}}, & \text{for Case 1 } (\tau \geq z_b) \\ \frac{P_{out,2}}{\Psi_{out,2}}, & \text{for Case 2 } (\tau \equiv \frac{1}{z_a}) \\ \frac{P_{out,3}}{\Psi_{out,3}}, & \text{for Case 3 } (\frac{1}{z_a} < \tau < z_b) \end{cases}
\]
where \( \Psi_{out,1}, \Psi_{out,2}, \) and \( \Psi_{out,3} \) are, respectively, given by
\[
\Psi_{out,1} = \sum_{p=0}^{K-1} \left( \begin{array}{c} K - 1 \end{array} \right) \frac{1}{p} \left( \frac{1}{z_a} \right)^{p-1} \tag{10}
\]
\[
\Psi_{out,2} = \sum_{p=0}^{K-1} \left( \begin{array}{c} K - 1 \end{array} \right) \frac{1}{p} \left( \frac{1}{z_a} \right)^{p-1} \tag{11}
\]
\[
\Psi_{out,3} = \frac{P_{out,1}}{\Psi_{out,1}} + \frac{P_{out,2}}{\Psi_{out,2}} \tag{12}
\]


\[
D(p,r,f,g) = \frac{K^{\text{out},3}}{(P \pi)} \left[ \sum_{m} \left( \begin{array}{l}
\mathbf{e}^{m} \\
\mathbf{e}^{m+1}
\end{array} \right) \right] \times \left[ \begin{array}{l}
\mathbf{e}^{m} \\
\mathbf{e}^{m+1}
\end{array} \right]
\]

(15)

\[
\mathbf{e}^{m} = \left( \begin{array}{c}
\sum_{r=0}^{\infty} \frac{r^{m}}{m!} \\
\sum_{r=0}^{\infty} \frac{r^{m+1}}{m!}
\end{array} \right)
\]

with \( m \) and \( r \) as defined in Section IV. Note that the diversity order does not depend upon the level of traffic asymmetry.

\[
P_{\text{out},3} = \frac{K^{\text{out},3}}{(P \pi)} \left[ \sum_{m} \left( \begin{array}{l}
\mathbf{e}^{m} \\
\mathbf{e}^{m+1}
\end{array} \right) \right] \times \left[ \begin{array}{l}
\mathbf{e}^{m} \\
\mathbf{e}^{m+1}
\end{array} \right]
\]

(16)

Proof: The proof is provided in Appendix B.

Furthermore, applying the approximation \( \Theta_{0,2} \approx \frac{1}{2} \) in (3) and (4) and the results into (16), we can recexpress in high SNR as

\[
P_{\text{out},3} \approx \frac{K^{\text{out},3}}{(P \pi)} \left[ \sum_{m} \left( \begin{array}{l}
\mathbf{e}^{m} \\
\mathbf{e}^{m+1}
\end{array} \right) \right] \times \left[ \begin{array}{l}
\mathbf{e}^{m} \\
\mathbf{e}^{m+1}
\end{array} \right]
\]

(17)

By considering different levels of asymmetry quantified by \( \Lambda_{1} \) and \( \Lambda_{2} \), we illustrate various SNR levels for Cases 1 and 2.

\[
\Theta_{0,2} \approx \frac{1}{2}
\]

Furthermore, applying the approximation \( \Theta_{0,2} \approx \frac{1}{2} \) in (3) and (4) and the results into (16), we can recexpress in high SNR as

\[
P_{\text{out},3} \approx \frac{K^{\text{out},3}}{(P \pi)} \left[ \sum_{m} \left( \begin{array}{l}
\mathbf{e}^{m} \\
\mathbf{e}^{m+1}
\end{array} \right) \right] \times \left[ \begin{array}{l}
\mathbf{e}^{m} \\
\mathbf{e}^{m+1}
\end{array} \right]
\]

(18)

and \( P_{\text{out},3} \) are given by

\[
P_{\text{out},3} = \frac{K^{\text{out},3}}{(P \pi)} \left[ \sum_{m} \left( \begin{array}{l}
\mathbf{e}^{m} \\
\mathbf{e}^{m+1}
\end{array} \right) \right] \times \left[ \begin{array}{l}
\mathbf{e}^{m} \\
\mathbf{e}^{m+1}
\end{array} \right]
\]

(19)

Furthermore, applying the approximation \( \Theta_{0,2} \approx \frac{1}{2} \) in (3) and (4) and the results into (16), we can recexpress in high SNR as

\[
P_{\text{out},3} \approx \frac{K^{\text{out},3}}{(P \pi)} \left[ \sum_{m} \left( \begin{array}{l}
\mathbf{e}^{m} \\
\mathbf{e}^{m+1}
\end{array} \right) \right] \times \left[ \begin{array}{l}
\mathbf{e}^{m} \\
\mathbf{e}^{m+1}
\end{array} \right]
\]

(20)

Furthermore, applying the approximation \( \Theta_{0,2} \approx \frac{1}{2} \) in (3) and (4) and the results into (16), we can recexpress in high SNR as

\[
P_{\text{out},3} \approx \frac{K^{\text{out},3}}{(P \pi)} \left[ \sum_{m} \left( \begin{array}{l}
\mathbf{e}^{m} \\
\mathbf{e}^{m+1}
\end{array} \right) \right] \times \left[ \begin{array}{l}
\mathbf{e}^{m} \\
\mathbf{e}^{m+1}
\end{array} \right]
\]

(21)

Furthermore, applying the approximation \( \Theta_{0,2} \approx \frac{1}{2} \) in (3) and (4) and the results into (16), we can recexpress in high SNR as

\[
P_{\text{out},3} \approx \frac{K^{\text{out},3}}{(P \pi)} \left[ \sum_{m} \left( \begin{array}{l}
\mathbf{e}^{m} \\
\mathbf{e}^{m+1}
\end{array} \right) \right] \times \left[ \begin{array}{l}
\mathbf{e}^{m} \\
\mathbf{e}^{m+1}
\end{array} \right]
\]

(22)
IV. OPA AND RELAY LOCATION

In this section, we optimize the power allocation and relay location to minimize the OOP of the considered system in the presence of asymmetric traffic conditions under Nakagami-\(m\) fading. This is motivated by the fact that the node distribution in practical wireless cellular networks may be random, and the transmitted power may be properly allocated. Moreover, the performance of the cellular multiuser TWR network is limited by the bottleneck link under system/channel parameters, and thereby, OPA and/or relay location can achieve a balance between the two connection over the relay, and hence the system fairness. Moreover, to identify key system/channel parameters and their effects on the system performance under asymmetric traffic conditions, we investigate both separate and joint optimization leading to the three problems, viz., OPA with fixed relay location, relay location with fixed power allocation, and jointly power allocation and relay location.

As the asymptotic expression in (17) is simple and found to be tight, we can use it to analyze the aforementioned optimization problems. For this, we adopt a path-loss model with exponent \(\nu\). We assume that the distance between \(T_a\) and \(R\) is \(d_a\), and the distance between \(R\) and \(T_b\) is \(d_b\). Consequently, we have \(\Omega_a = d_a^{-\nu}\) and \(\Omega_b = d_b^{-\nu}\). Furthermore, we consider a linear relaying model \(d_a + d_b = 1, d_a = d,\) and \(d_b = 1 - d\) for \(d \in (0, 1)\). By setting these assumptions in (17) and after some simplifications, we can reexpress \(\mathcal{P}_{\text{out}}\) in (17) as presented in (18) at the bottom of this page with \(\omega_a = \frac{(m_a N_a)^{1+n}}{m_a N_a}\) and \(\omega_b = \frac{(m_b N_b)^{1+n}}{m_b N_b}\).

A. OPA Under Fixed Relay Location

For the considered system under traffic asymmetry, the outage performance can be improved by optimally allocating the total power to each terminal in the network. In what follows, we present two power allocation optimization problems to minimize the OOP when relay location is fixed: 1) OPA I: power allocation at all terminals and 2) OPA II: power allocation at two end terminals, while relay terminal is assigned with half of the total power.

1) OPA I: With a total power constraint (TPC)\(^1\) \(P_a + P_b + P_r = P_t\) of the considered system, we investigate the power allocation at all terminals to minimize the OOP. The optimization problem can be formulated as

\[
P^*_a, P^*_b, P^*_r = \arg \min_{P_a, P_b, P_r} \mathcal{P}_{\text{out}}
\]

subject to \(P_a + P_b + P_r - P_t \leq 0\) and \(P_a, P_b, P_r > 0\). (19)

Subsequently, three optimization subproblems can be formulated pertaining to the three cases in (18).

\[
\begin{aligned}
\mathcal{P}_{\text{out}} \approx_{\nu \to \infty} & \left\{ \begin{array}{ll}
\omega_a \left( \frac{\Lambda_{bh}(P_b + P_r)}{P_r} \right)^{m_a N_a} d^{P_b N_a} + \omega_b \left( \frac{\Lambda_{bh}(P_a + P_r)}{P_b} \right)^{m_b N_a} \\
\omega_a \left( \frac{\Lambda_{bh}(P_b + P_r)}{P_r} \right)^{m_a N_a} d^{P_b N_a} + \omega_b \left( \frac{\Lambda_{bh}(P_a + P_r)}{P_b} \right)^{m_b N_a} \\
\omega_a \left( \frac{\Lambda_{bh}(P_b + P_r)}{P_r} \right)^{m_a N_a} d^{P_b N_a} + \omega_b \left( \frac{\Lambda_{bh}(P_a + P_r)}{P_b} \right)^{m_b N_a}
\end{array} \right. \\
(1 - d)^{\nu m_a K} \leq S_1, \quad & \text{for Case 1} \\
(1 - d)^{\nu m_b K} \leq S_2, \quad & \text{for Case 2} \\
(1 - d)^{\nu m_b K} \leq S_3, \quad & \text{for Case 3}
\end{aligned}
\]

Case 1: When \(S_1\) of (18) is selected as the objective function, the optimization problem in (19) can be formulated as

\[
P^*_a, P^*_b, P^*_r = \arg \min_{P_a, P_b, P_r} \mathcal{S}_1
\]

subject to \(P_a + P_r - \frac{\tau P_t}{1 + \tau} \leq 0\)

\[
P_a + P_b + P_r - P_t \leq 0 \quad \text{and} \quad P_a, P_b, P_r > 0
\]

(20)

where the inequality constraint \(P_a + P_r \leq \frac{\tau P_t}{1 + \tau}\) is transformed from the condition \(\tau \geq \frac{d}{b}\). It seems difficult to get the OPA by solving the optimization problem given in (20) directly. Therefore, by applying the Karush–Kuhn–Tucker (KKT) conditions for Lagrangian optimality [14], the optimal values of \(P_a, P_b, P_r\) for Case 1 can be solved with the help of following equations:

\[
\begin{aligned}
\omega_a m_a N_a \left( \frac{\Lambda_{bh}(P_b - P_r)}{P_r} \right)^{m_a N_a} - \omega_b m_b K \left( \frac{P_b P_r}{P_a P_t} \right) - 1 &= 0 \\
- \omega_b m_b K \left( \frac{\Lambda_{bh}(1-d)^{\nu}}{P_r} \right)^{m_b K} &= 0 \\
P_a + P_r - \frac{P_t}{1 + \tau} &= 0 \\
P_a + P_b + P_r - P_t &= 0.
\end{aligned}
\]

From (21), \(P^*_a, P^*_b, P^*_r\), and \(P^*_r\) for various values of \(m_a, m_b, N_a, N_b,\) and \(K\) can be evaluated by using standard numerical methods.

Case 2: If \(S_2\) of (18) is considered as the objective function, the optimization problem in (19) can be posed as

\[
P^*_a, P^*_b, P^*_r = \arg \min_{P_a, P_b, P_r} \mathcal{S}_2
\]

subject to \(P_a + P_r - \frac{P_t}{1 + \tau} \leq 0\)

\[
P_a + P_b + P_r - P_t \leq 0 \quad \text{and} \quad P_a, P_b, P_r > 0
\]

(22)

where the inequality constraint \(P_b + P_r \leq \frac{P_t}{1 + \tau}\) is transformed from the condition \(\tau \leq \frac{d}{b}\). The optimization problem presented in (22) can be evaluated by following the similar approach as adopted to obtain (21). Consequently, we can get \(P^*_a, P^*_b, P^*_r\) for Case 2 by solving the following equations with the help of standard root finding algorithms:
Case 3: When $S_3$ of (18) is used as the objective function, the optimization problem in (19) can be stated as

$$
P_a^*, P_b^*, P_r^* = \arg \min_{P_a, P_b, P_r} S_3
$$

subject to $P_a + P_r - \frac{\tau P_1}{1 + \tau} \geq \varrho_1 > 0$

$P_b + P_r - \frac{P_1}{1 + \tau} \geq \varrho_2 > 0$

$P_a + P_b + P_r - P_1 \leq 0$ and $P_a, P_b, P_r > 0$ \hspace{1cm} (24)

where the inequality constraint $\varrho_1 > 0$ and $\varrho_2 > 0$ are transformed from the condition $\frac{1}{\tau} < \theta < \zeta_b$. Due to complicity of the objective function and constraints presented in (24), it is tedious to obtain the optimal power allocation. Therefore, by ignoring the first two constraints in (24) and applying KKT conditions for Lagrangian optimality, the corresponding optimization problem can be solved to get

$$
\begin{align*}
\omega_a m_a N_a \left( \frac{\lambda_{a_1} - \mu_a N_a}{P_a} \right) & - \omega_a m_a \left( \frac{\lambda_{a_2} - \nu_a N_a}{P_a} \right) = 0 \\
\omega_b m_b K \left( \frac{\lambda_{b_1} - \mu_b N_b}{P_b} \right) & - \omega_b m_b \left( \frac{\lambda_{b_2} - \nu_b N_b}{P_b} \right) = 0 \\
\omega_{a_1} m_{a_1} N_{a_1} \left( \frac{\lambda_{a_1} - \mu_{a_1} N_{a_1}}{P_{a_1}} \right) & - \omega_{a_1} m_{a_1} \left( \frac{\lambda_{a_2} - \nu_{a_1} N_{a_1}}{P_{a_1}} \right) = 0 \\
\omega_{b_1} m_{b_1} K \left( \frac{\lambda_{b_1} - \mu_{b_1} K}{P_{b_1}} \right) & - \omega_{b_1} m_{b_1} \left( \frac{\lambda_{b_2} - \nu_{b_1} K}{P_{b_1}} \right) = 0
\end{align*}
$$

(25)

In particular, when $m_a = m_b = 1$, $N_a = K = 1$, $\lambda_{a_1} = \lambda_{b_1} = \lambda_{a_2} = \lambda_{b_2}$, and $d = 1/2$, the solution of (25) reduces to $P_a = \frac{P_1}{2 + \sqrt{2}}$, $P_b = \frac{P_1}{2 + \sqrt{2}}$, and $P_r^* = \frac{P_1}{2 + \sqrt{2}}$. For other values of $m_a, m_b, N_a, N_b, K$, one can solve (25) by applying numerical root finding algorithms to obtain $P_a^*, P_b^*$, and $P_r^*$.

Moreover, it can be observed that if $\varrho_1 > 0$ and $\varrho_2 > 0$, the optimal solutions of $P_a$, $P_b$, and $P_r$ for Case 3 is (25), otherwise, it is on the boundary $\varrho_1 = 0$ or $\varrho_2 = 0$. When $\varrho_1 = 0$, it can be proven that the optimal solution for Case 3 is equivalent to the optimal solution given in (21). Likewise, for $\varrho_2 = 0$, the OPA for Case 3 is equivalent to (23). Therefore, based on aforesaid analysis, we present an OPA algorithm to optimize the overall outage performance, which can be depicted as follows:

$$
P_a^*, P_b^*, P_r^* = \begin{cases} 
(25), & \varrho_1 > 0 \text{ and } \varrho_2 > 0 \\
(21) \text{ or } (23), & \text{otherwise.}
\end{cases}
$$

(26)

2) OPA II: It can be shown, as in [19], that allocating half of the total power to the relay minimizes the OOP for the considered system, i.e., $(P_a^* + P_b^*) = \frac{P_1}{2}$, irrespective of channel/system parameters. Therefore, making use of this result and (18), we can reformulate (19) as

$$
P_a^*, P_b^* = \arg \min_{P_a, P_b} \tilde{P}_{out}
$$

subject to $P_a + P_b - \frac{P_1}{2} \leq 0$ and $P_a, P_b > 0$ \hspace{1cm} (27)

where $\tilde{P}_{out}$ is given in (28), shown at the bottom of the next page. Subsequently, three optimization subproblems can be formulated pertaining to the three cases in (28) in the sequel.

Case 1: When $X_1$ of (28) is considered as the objective function, the problem in (27) for Case 1 can be posed as

$$
P_a^*, P_b^* = \arg \min_{P_a, P_b} X_1
$$

subject to $P_a - \frac{P_1}{2} \left( \frac{\tau - 1}{\tau + 1} \right) \leq 0$

$P_a + P_b - \frac{P_1}{2} \leq 0$ and $P_a, P_b > 0$. \hspace{1cm} (29)

The optimization problem presented in (29) can be evaluated by applying the KKT conditions for Lagrangian optimality as used previously. Consequently, we can get $P_a^*$ and $P_b^*$ as

$$
P_a^* = \frac{P_1}{2} \left( \frac{\tau - 1}{\tau + 1} \right) \text{ and } P_b^* = \frac{P_1}{\tau + 1}. \hspace{1cm} (30)
$$

Case 2: If $X_2$ of (28) is selected as the objective function, the optimization problem for Case 2 is given by

$$
P_a^*, P_b^* = \arg \min_{P_a, P_b} X_2
$$

subject to $P_b - \frac{P_1}{2} \left( \frac{1 - \tau}{\tau + 1} \right) \leq 0$

$P_a + P_b - \frac{P_1}{2} \leq 0$ and $P_a, P_b > 0$. \hspace{1cm} (31)

The problem in (31) can be solved by following the similar steps as used to obtain (30), and hence, we get

$$
P_a^* = \frac{P_1}{1 + \tau} \text{ and } P_b^* = \frac{P_1}{2} \left( \frac{1 - \tau}{1 + \tau} \right). \hspace{1cm} (32)
$$

Case 3: When $X_3$ of (28) is used as the objective function, the optimization problem in (27) for Case 3 can be stated as

$$
P_a^*, P_b^* = \arg \min_{P_a, P_b} X_3
$$

subject to $P_a - \frac{P_1}{2} \left( \frac{\tau - 1}{\tau + 1} \right) \geq \varrho_1 > 0$

$P_b - \frac{P_1}{2} \left( \frac{1 - \tau}{1 + \tau} \right) \geq \varrho_2 > 0$

$P_a + P_b - \frac{P_1}{2} \leq 0$ and $P_a, P_b > 0$. \hspace{1cm} (33)

The optimization problem in (33) can be solved by following the same approach as used above to get

$$
\begin{align*}
\omega_a m_a N_a \left( 2 \lambda_{a_1} - \mu_a N_a \right) & - \omega_a m_a \left( \lambda_{a_2} - \nu_a N_a \right) \left( P_1 - P_a \right) m_a N_a - 1 = 0 \\
\omega_b m_b K \left( 2 \lambda_{b_1} - \mu_b K \right) & - \omega_b m_b \left( \lambda_{b_2} - \nu_b K \right) \left( P_1 - P_b \right) m_b K - 1 = 0
\end{align*}
$$

(34)

The optimal values of $P_a$ and $P_b$ can be obtained by solving the equations in (34) using standard root finding algorithms. For the special case, when $m_a = m_b = 1$, $N_a = K = 1$, $\lambda_{a_1} = \lambda_{b_1} = \lambda_{a_2} = \lambda_{b_2}$, and $d = 1/2$, the optimal solution of (34) yields $P_a^* = \frac{P_1}{4}$ and $P_b^* = \frac{P_1}{4}$. Moreover, we can see that $P_a^*$ and $P_b^*$ are given by (34) when $\varrho_1 > 0$ and $\varrho_2 > 0$ are satisfied, otherwise, it is on the boundary $\varrho_1 = 0$ or $\varrho_2 = 0$. When $\varrho_1 = 0$, it is proven that the $P_a^*$ and $P_b^*$ for Case 3 are same as in (30). Likewise, for $\varrho_2 = 0$. 

\clearpage
the $P_a^*$ and $P_r^*$ for Case 3 are equivalent to (32). Therefore, we can present a hybrid scheme to implement the OPA to minimize the OOP, and can be summarized as follows:

$$P_a^*, P_r^* = \begin{cases} (34), & \tilde{\vartheta}_1 > 0 \text{ and } \tilde{\vartheta}_2 > 0 \\ (30) \text{ or } (32), & \text{otherwise.} \end{cases}$$

(35)

It is also noted from OPA I and OPA II that under unbalanced conditions (i.e., $R_{th_a} \neq R_{th_b}$ and/or $m_a N_a \neq m_b K$), the system fairness can be maintained by properly allocating the powers at each terminal, as shown numerically in Section V.

B. Optimal Relay Location Under Fixed Power Allocation

For fixed power allocation ($P_a, P_b, P_r$), the optimization problem for the optimal relay location to minimize the OOP can be posed using (18) as

$$d^* = \arg \min_d \mathcal{P}_{\text{out}}$$

subject to $0 < d < 1$. (36)

From (18), we infer that the second derivative of $\mathcal{P}_{\text{out}}$ is strictly a convex function in the range $d \in (0, 1)$. Consequently, by equating its first derivative with respect to $d$ to zero, we obtain

$$\omega_a m_a N_a \left( \frac{\Lambda_{th_a} (P_b + P_r)}{P_a} \right)^{m_a N_a} d^{\nu_{m_a} N_a - 1} = \omega_b m_b K \left( \frac{\Lambda_{th_b} P_a}{P_b} \right)^{m_b N_b} - (1 - d)^\nu_{m_b} K^{-1}, \text{ for Case 1}$$

(37)

$$\omega_a m_a N_a \left( \frac{\Lambda_{th_a} (P_b + P_r)}{P_a} \right)^{m_a N_a} d^{\nu_{m_a} N_a - 1} = \omega_b m_b K \left( \frac{\Lambda_{th_b} P_a}{P_b} \right)^{m_b N_b} - (1 - d)^\nu_{m_b} K^{-1}, \text{ for Case 2}$$

(38)

$$\omega_a m_a N_a \left( \frac{\Lambda_{th_a} (P_b + P_r)}{P_a} \right)^{m_a N_a} d^{\nu_{m_a} N_a - 1} = \omega_b m_b K \left( \frac{\Lambda_{th_b} P_a}{P_b} \right)^{m_b N_b} - (1 - d)^\nu_{m_b} K^{-1}, \text{ for Case 3.}$$

(39)

In particular, for $m_a = m_b = 1$ and $N_a = K = 1$, the optimal relay location from (37)–(39) reduces to $d^* = \left[ 1 + \left( \frac{P_a + P_r}{P_b} \right)^{\tau_{m_a} N_a} \right]^{-1}$ for Case 1, $d^* = \left[ 1 + \left( \frac{P_a + P_r}{P_b + P_r} \right)^{\tau_{m_a} N_a} \right]^{-1}$ for Case 2, and $d^* = \left[ 1 + \left( \frac{P_a + P_r}{P_b + P_r} \right)^{\tau_{m_a} N_a} \right]^{-1}$ for Case 3, respectively. For other values of $m_a, m_b, N_a, \text{ and } K$, the $d^*$ can be evaluated from (37)–(39) via numerical root finding algorithms such as Newton–Raphson and Bisection methods. Note that under unbalanced conditions (i.e., $R_{th_a} \neq R_{th_b}$ and/or $m_a N_a \neq m_b K$), the relay should be placed closer to either end terminal in order to achieve the system fairness, as also demonstrated numerically in Section V.

C. Jointly OPA and Relay Location

Since power allocation and relay location are two key factors that have a significant impact on the system performance, studying their joint optimization is more challenging than scenarios where the optimization of these factors are conducted separately. Therefore, it is quite reasonable to jointly optimize the power allocation and relay location to minimize the OOP. Specifically, we study two joint optimization problems corresponding to the two power allocation schemes (i.e., OPA I and OPA II) in the sequel.

1) Joint Optimization of Power Allocation (OPA I) and Relay Location: The joint optimization problem under three cases based on a traffic asymmetry can be posed using (18) as

$$P_a^*, P_b^*, P_r^*, d^* = \arg \min_{P_a, P_b, P_r, d} \mathcal{P}_{\text{out}}$$

subject to $P_a + P_b + P_r \geq 0$ and $0 < d < 1$. (40)

By differentiating the objective function in (18) twice with respect to $P_a, P_b, \text{ and } d$, we can show that the Hessian matrix is positive definite in the intervals $P_a, P_b, P_r \in (0, P_t)$ and $d \in (0, 1)$, Therefore, by applying the KKT conditions and after some algebraic manipulations, we obtain

$$\begin{align*}
P_a &= \frac{P_t}{4} \left[ 1 + \sqrt{\left( \frac{1 + d}{4} \right)^2 - \frac{4d}{1 + d}} \right] \\
P_b &= P_t \left[ 1 - \frac{\tau_{m_a} N_a}{P_a} \right] \\
P_r &= P_t - P_a - P_b. \\
\end{align*}$$

(41)

$$\begin{align*}
P_a &= \frac{P_t}{4} \left[ 2 - \sqrt{\left( 2 - \frac{d}{4} \right)^2 - \frac{4(1-d)}{1 + d}} \right] \\
P_b &= P_t \left[ 1 - \frac{\tau_{m_a} N_a}{P_a} \right] \\
P_r &= P_t - P_a - P_b. \\
\end{align*}$$

(42)

$$\begin{align*}
P_a &= \frac{P_t}{4} \left[ 2d + 1 - \sqrt{4d + 1 - 4d^2} \right] \\
P_b &= (P_t - P_a) \left[ 1 - \frac{\tau_{m_a} N_a}{P_a} \right] \\
P_r &= P_t - P_a - P_b. \\
\end{align*}$$

(43)

Substituting the result presented in (41) for Case 1 into (37) and solving with standard root finding algorithms, we can find the optimal value of $d$ and then corresponding OPA from (21) accordingly. Likewise, the $P_a^*, P_b^*, P_r^*$, and $d^*$ for Case 2 can be obtained using (42), (38), and (23), and for Case 3 using (43), (39), and (25). Moreover, the results obtained for Case 3 in (43) are only valid when constraints $P_a + P_r > \frac{\tau_{m_a} N_a}{P_a} > 0$ and $P_b + P_r > \frac{\tau_{m_a} N_a}{P_a} > 0$ are satisfied, otherwise, it is on the boundary.

$$\bar{P}_{\text{out}} \sim \mathcal{N} \text{ for } P_a, P_b, P_r, P_t, d$$

(28)
\[ P_a + P_r - \frac{P_a}{1 + \tau} = 0 \text{ or } P_b + P_r - \frac{P_b}{1 + \tau} = 0. \]

Therefore, based on the aforementioned analysis, we can present a joint relay location and power allocation algorithm to minimize the OOP, which can be depicted as follows:

\[
P_{a}^{*}, P_{b}^{*}, d^{*} = \begin{cases} 
(43), (39), (25), & \text{if } P_a + P_r - \frac{P_a}{1 + \tau} > 0 \\
\text{and } P_b + P_r - \frac{P_b}{1 + \tau} > 0 & \text{if } ((41), (37), (21)) \text{ or } ((42), (38), (23)) \\
\text{otherwise.} & \end{cases}
\]

2) Joint Optimization of Power Allocation (OPA II) and Relay Location: The joint optimization problem under three cases based on traffic asymmetry can be posed using (28) as

\[ P_{a}^{*}, P_{b}^{*}, d^{*} = \arg \min_{P_a, P_b} \overline{P}_\text{out} \text{ subject to } P_a + P_b - \frac{P_t}{2} \leq 0 \]

\[ P_a, P_b > 0 \text{ and } 0 < d < 1. \]  

\[ (45) \]

It can be shown that the matrix of the second partial derivative of \( \overline{P}_\text{out} \) in (28) is positive definite in the intervals \( P_a, P_b \in (0, P_t/2) \), and \( d \in (0, 1) \). Therefore, by applying the KKT conditions for Lagrangian optimality and after some simplifications, we obtain

\[ \begin{align*}
\omega_a m_a N_a \left( \frac{P_a (2P_a + P_t)}{P_r} \right)^{\frac{m_a N_a}{m_a K}} & d^{m_a N_a - 1} \\
\omega_b m_b K \left( \frac{P_a (2P_a + P_t)}{P_r} \right)^{m_b K} & (1 - d)^{m_b K - 1} \\
\frac{P_a}{2} & = 1 - \tau - d^{1 + \frac{2 d - 3 d^2 + 1}{2 d - 1}} \\
\frac{P_b}{2} & = \frac{1}{2} - P_a \\
\end{align*} \]  

\[ (46) \]

\[ (47) \]

\[ (48) \]

It can be readily observed that the jointly optimal values of \( P_a, P_b, \) and \( d \) under Cases 1 and 2 (with \( P_r = \frac{P_t}{2} \)) can be obtained by evaluating (46) and (47), respectively. For Case 3, substituting the result given in (48) into (39) with \( P_t = \frac{P_t}{2} \) and solving via numerical methods, we can find the optimal value of \( d \) and then the corresponding OPA from (34) accordingly. Moreover, (48) is only valid if the constraints \( P_a - \frac{P_t}{2} \left( \frac{1}{1 + \tau} \right) > 0 \) and \( P_b - \frac{P_t}{2} \left( \frac{1}{1 + \tau} \right) > 0 \) are satisfied, else, it is on the boundary \( P_a - \frac{P_t}{2} \left( \frac{1}{1 + \tau} \right) = 0 \) or \( P_b - \frac{P_t}{2} \left( \frac{1}{1 + \tau} \right) = 0 \). Thus, we can present a hybrid scheme to implement the jointly optimal relay location and power allocation to minimize the OOP, which can be described as follows:

\[ P_{a}^{*}, P_{b}^{*}, d^{*} = \begin{cases} 
(48), (39), (34), & \text{if } P_a - \frac{P_t}{2} \left( \frac{1}{1 + \tau} \right) > 0 \\
\text{and } P_b - \frac{P_t}{2} \left( \frac{1}{1 + \tau} \right) > 0 & \text{if } (46) \text{ or } (47), \text{ otherwise.} \\
\end{cases}
\]  

\[ (49) \]

V. NUMERICAL RESULTS AND DISCUSSIONS

In this section, we provide numerical and simulation results to validate our theoretical analysis for the considered system. First, we set Case 1 (\( R_{th} = 1 \) b/s/Hz, \( R_{th} = 0.5 \) b/s/Hz), Case 2 (\( R_{th} = 0.5 \) b/s/Hz, \( R_{th} = 1 \) b/s/Hz), and Case 3 (\( R_{th} = 0.5 \) b/s/Hz, \( R_{th} = 0.5 \) b/s/Hz). We adopt the path-loss model with exponent \( \nu = 4 \), and considered the normalized distances \( d_a = d \) and \( d_b = 1 - d \) with \( d \in (0, 1) \). As such, \( \Omega_a = d^{-4} \) and \( \Omega_b = (1 - d)^{-4} \).

For Figs. 2 and 3, we assume \( d = 0.5 \) and \( P_a = P_b = P_r = P_t/3 \) [uniform power allocation (UPA)], and considered \( \frac{P_r}{P_t} \) as the transmit SNR. Afterwards, in Figs. 4–7, we perform the experiments for different relay locations. Moreover, the results shown are averaged over 10\(^6\) independent trials.

Figs. 2 and 3 depict the OOP performance of the considered system versus SNR under three cases for various values of fading severity parameters (\( m_a \) and \( m_b \)), number of antennas (\( N_a \)), and number of users (\( K \)). It can be observed from these figures that...
Fig. 4. Impact of relay location and power allocation on the OOP performance under Cases 1 and 2.

Fig. 5. Impact of relay location and power allocation on the OOP performance under Case 3.

Fig. 6. Optimized transmission powers versus $d$ of the considered OPA schemes under Cases 1 and 2.

the analytical results\(^2\) using (9) are matching perfectly with the simulation results, and the asymptotic curves using (17) are very close to the exact results in the medium-to-high SNR regime. For asymmetric traffic (Case 1), we can see from Fig. 2 that the OOP performance improves significantly when $m_a > m_b$ and/or $N_a > K$. For instance, when $m_a = 2 > m_b = 1$ (with $N_a = K = 1$) the OOP performance is better than that of the scenario when $m_a = 1 < m_b = 2$ (with $N_a = K = 1$). This is expected because the system outage is dominated by one-way traffic $T_a \rightarrow R \rightarrow T_b\,*$ with weaker $T_a \rightarrow R$ link. Whereas, for Case 2, the system outage is dominated by one-way traffic $T_a \leftarrow R \leftarrow T_b\,*$ with bottleneck $R \leftarrow T_b\,*$ link, and hence the OOP performance can be improved when $m_b > m_a$ and/or $K > N_a$. For Case 3, the curves in Fig. 3 demonstrate that the OOP performance improves as $N_a$ and $K$ increases and/or fading severity over both hops decreases. We can also observe that the slope of the OOP curves is same for $N_a = 1, 2$ and $m_b = 1$ when $m_a = K = 1$. This is owing to the limiting behavior of the considered protocol where the performance is dominated by the weaker hop ($m_b = K = 1$). Moreover, the diversity order $\min(m_a N_a, m_b K)$ is verified for various scenarios under three cases (see Figs. 2 and 3).

In Figs. 4 and 5, we demonstrate the impact of relay location and power allocation on the system performance in the presence of different rate requirements when SNR = 20 dB. For asymmetric traffic (Cases 1 and 2), it can be observed from Fig. 4 that the system performance is limited by the weaker hop. Therefore, a shorter link will improve the quality of weaker hop, and hence the OOP performance. For instance, in Case 1 ($R_{th_1} = 0.5$ b/s/Hz, $R_{th_2} = 1$ b/s/Hz) when $m_a = m_b = 1$ and $N_a = K = 1$, the OOP performance can be improved with $d^* = 0.44$. It can also

\(^2\)Note that the outage expression for Case 3 involves infinite series expansion which converges quickly as shown in Table II. The Table also shows that first three or four terms are sufficient to achieve the convergence value.
be seen from Fig. 4 that the OPA II has same performance as the UPA when the relay is closed to the terminal with higher target rate, and performs better at other relay locations. Moreover, the OPA I outperforms UPA, irrespective of the relay locations. Furthermore, the OPA I performs better than the OPA II for different relay locations except at the relay location where both coincide. This implies that the OPA I plays a more significant role in the OOP performance improvement compared to the OPA II and UPA. Albeit, the OOP performance can be further improved by choosing jointly OPA and relay location, especially under unbalanced conditions ($R_{th1} \neq R_{th2}$ and $m_a N_a \neq m_b K$).

For Case 3, it can be seen from Fig. 5 that the minimum OOP occurs at $d^* = 0.5$ when $R_{th1} = R_{th2} = 0.5$ b/s/Hz, $m_a = m_b = 1$, and $N_a = K = 1$ with UPA. However, when $m_a, m_b, N_a$, and $K$ increases, the OOP performance can be significantly improved by choosing the relay location optimally. For instance, when $m_a = 2, m_b = 1, N_a = 1, K = 3$, and UPA, the system performance is limited by the weaker $T_a - R$ link. In this case, a shorter $T_a - R$ link ($d^* = 0.33$) will improve the link quality, and thereby the OOP performance. Moreover, it is shown that the OPA I outperforms OPA II and UPA, irrespective of the relay locations (see Table III when $m_a = m_b = 1$ and $N_a = K = 1$). This indicates that the OPA I is more efficient and can achieve a significant OOP performance compared to the OPA II and UPA. Such a performance gain is expected due the fact that the OPA I distributes the total power ($P_I$) at each terminal in appropriate proportion according to the different traffic requirements and/or system/channel parameters, while OPA II only allocates $\frac{P}{2}$ to the end terminals, as the relay power is fixed with $P_R^* = \frac{P}{2}$. More importantly, the OOP performance can be further improved by jointly optimizing the relay location and power allocation, particularly under unbalanced conditions. For instance, when $m_a = 2, m_b = 1, N_a = 1$, and $K = 3$, the $P_{out} = 8.82 \times 10^{-7}$ at $d^* = 0.065$ with OPA I and $P_{out} = 1.62 \times 10^{-6}$ at $d^* = 0.20$ with OPA II. This suggests that the joint optimization of relay location and OPA I has more noticeable impact on the OOP performance than that of the joint optimization of relay location and OPA II.

In Figs. 6 and 7, we plot the optimized power levels versus $d$ for the two power control schemes (OPA I and OPA II) corresponding to the Figs. 4 and 5, respectively, when $m_a = m_b = 1$ and $N_a = K = 1$. It is observed that all the optimized power levels satisfy the TPC $P_I$, confirming the correctness of our derived analytical expressions. As expected, for asymmetric traffic (Case I) in Fig. 6, the power at terminal $T_{b1}$ under OPA I and OPA II is fixed at certain level, regardless of the relay location. This is due to the one-sided traffic flow $T_a \rightarrow R \rightarrow T_{b1}$. Moreover, in contrast to OPA II where $P_a$ and $P_r$ are fixed at certain power levels, OPA I optimally allocates $P_a$ and $P_r$ for all relay locations. Therefore, with OPA I, it is possible that either $T_a$ or $R$ may use a lower transmit power to achieve a better OOP performance. The description of Case 2 ($T_a \leftarrow R \leftarrow T_{b2}$) in Fig. 6 can be made similar to Case 1, and omitted for the sake of brevity. For Case 3, it can be observed from Fig. 7 that there is noticeable gap in the optimal relay power ($P_{r}^*$) of the two OPA schemes. This shows that the OPA I can achieve a better OOP performance than OPA II by allocating relatively less relay power under different relay locations (e.g., when $m_a = m_b = 1$, $N_a = K = 1$, and $d = 0.1$, the $P_{r}^* = 11.2828$ dBW under OPA I and $P_{r}^* = 16.9897$ dBW under OPA II), as shown in Fig. 5.

VI. CONCLUSION

We have evaluated the OOP performance of the cellular multiuser TWR network in the presence of traffic asymmetry under Nakagami-m fading. Through the analytical results, we highlighted the impact of traffic asymmetry on the OOP performance, and some useful insights are also achieved. We have shown that the OOP can be deduced by a one-way traffic flow for some asymmetric traffic. Specifically, the OOP is determined only by the link $T_a \rightarrow R \rightarrow T_{b1}, (T_a \leftarrow R \leftarrow T_{b2})$ for all $\tau \geq z_b (\tau \geq \frac{1}{2})$. Moreover, to identify the impact of key system/channel parameters, we have investigated the optimization problems of power allocation (i.e., OPA I and OPA II) and relay location (including joint optimization) to minimize the OOP. The numerical and simulation results validated our theoretical findings, and highlighted that under highly asymmetric traffic with different channel/system parameters, the OPA I outperforms OPA II in terms of OOP for various relay locations, except at the relay location where both coincide. Whereas, under symmetric traffic, the OPA I exhibits better performance than OPA II, irrespective of the relay locations and/or channel/system parameters. Our results also demonstrated that the OOP performance can be further improved if the power allocation and relay location are jointly optimized, especially under unbalanced conditions. These results therefore provide useful design guidelines for the practical traffic-aware cellular TWR networks.

In the future, various possible extensions may deserve for consideration: 1) other performance measures such as average symbol error probability and ergodic sum rate can be derived and 2) comprehensive performance evaluation and optimization for the cellular multiuser TWR networks by incorporating multiple relays with relay-user selection schemes.

APPENDIX A

PROOF OF (9)

For Case 1, the OOP can be expressed using (8) as

\[
P_{out,1} = L_1 = \frac{\Lambda_{i1}}{\Lambda_{i1}} < \frac{\Theta_{th1} \Lambda_{i1}}{\Theta_{th1} |\Lambda_{i1}|}\]

\[
\int_{0}^{\infty} f_{\Lambda_{i1}}(y) \int_{0}^{\infty} f_{\Lambda_{i1}}(x) dx dy
\]

\[
\delta_{F1} \int_{0}^{\infty} f_{\Lambda_{i1}}(y) \int_{0}^{\infty} f_{\Lambda_{i1}}(x) dx dy.
\]

By invoking the PDFs of $\Lambda_{i1}$ and $\Lambda_{i1}$ from (5) and (6), respectively, into (51), the integral $I_1$ can be simplified using [24, eq.
(8.350.1) as

\[
I_1 = \frac{K}{\Gamma(m_b)} \left( \frac{m_b}{\Lambda_{1b}} \right) \sum_{p=0}^{m_b-1} \sum_{r=0}^{K-1} \left( \frac{K-1}{p} \right) (-1)^p b^p \\
\times \left( \frac{\Lambda_{1b}}{m_r(p+1)} \right)^{m_b+r} \Gamma \left( m_b + r, \frac{m_b(p+1)\Theta_{th}}{\Lambda_{1b}} \right).
\]

(52)

Similarly, the integral \( I_2 \) in (51) can be evaluated with the aid of [24, eqs. (8.350.1) and (8.352.6)] as

\[
I_2 = \frac{K}{\Gamma(m_b)} \left( \frac{m_b}{\Lambda_{1b}} \right) \sum_{p=0}^{m_b-1} \sum_{r=0}^{K-1} \left( \frac{K-1}{p} \right) (-1)^p b^p \\
\times \int_{\Theta_{th}}^{\infty} y^{m_b+r-1} e^{-\frac{m_b}{\Lambda_{1b}} y} dy - \int_{\Theta_{th}}^{\infty} y^{m_b+r-1} e^{-\frac{m_b}{\Lambda_{1b}} y} dy \\
\times \int_{\Theta_{th}}^{\infty} y^{m_b+r-1} e^{-\frac{m_b}{\Lambda_{1b}} y} dy
\]

where \( J_1 \) in (53) can be readily obtained using [24, eq. (8.350.2)], and \( J_2 \) can be evaluated by applying transformation of variable \( z = y - \Theta_{th} \), and then using [24, eq. (3.471.9)]. Consequently, invoking these into (53) and the result along with (52) into (51), we can get \( P_{out} \) for Case 1 in (10).

Likewise, \( P_{out} \) for Case 2 \( (\tau \leq \frac{1}{2}) \) in (11) can be obtained and represented by replacing \( \Theta_{th} \) with \( \Theta_{th_2} \), \( \Lambda_{1a} \) and \( \Lambda_{1b} \), with \( \Lambda_{2a} \) and \( \Lambda_{2b} \), respectively, in (10).

For Case 3, the OOP can be expressed using (8) as

\[
P_{out,3} = L_3
\]

\[
= \Pr \left[ \frac{\Lambda_{1a} \Lambda_{1b}}{\Lambda_{1a} + \Lambda_{1b}} < \Theta_{th_1}, \frac{\Lambda_{1a} \Lambda_{1b}}{\Lambda_{1a} + \Lambda_{1b}} \right] + \Pr \left[ \frac{\Lambda_{2a} \Lambda_{2b}}{\Lambda_{2a} + \Lambda_{2b}} < \Theta_{th_2}, \frac{\Lambda_{2a} \Lambda_{2b}}{\Lambda_{2a} + \Lambda_{2b}} \right].
\]

(54)

The \( P_{out,3}^{(1)} \) can be obtained with tedious mathematical steps (skipped for brevity) with the help of [24, eqs. (8.350.1) and (8.352.6)], Taylor series expansion, and [25, eq. (5.1.4)], as given in (13). Following similar approach as used above, we can get \( P_{out,3}^{(2)} \) in (14). Then, using (13) and (14) into (12), we can get the desired expression of \( P_{out} \) for Case 3.

**APPENDIX B**

**PROOF OF (16)**

To obtain the asymptotic behavior of \( P_{out} \) for Case 1 in (16), we first simplify the \( P_{out} \) for Case 1 in (50) at high SNR with

\[
\lim_{\Lambda_{1a} \rightarrow \infty, \Lambda_{1b} \rightarrow \Theta_{th_1}} \Theta_{th_1} = \Theta_{th_2}
\]

\[
P_{out} \approx 1 - \Pr \left[ \frac{\Lambda_{1a} \Theta_{th_1}}{\Lambda_{1b} \Theta_{th_1}} \right] = F_{\Lambda_{1a}}(\Theta_{th_1}) + F_{\Lambda_{1b}}(\Theta_{th_1}) - F_{\Lambda_{1a}}(\Theta_{th_1}) F_{\Lambda_{1b}}(\Theta_{th_1}).
\]

(55)

Furthermore, making use of the fact that \( F_{\Lambda_{1a}}(\Theta_{th_1}) + F_{\Lambda_{1b}}(\Theta_{th_1}) \geq F_{\Lambda_{1a}}(\Theta_{th_1}) F_{\Lambda_{1b}}(\Theta_{th_1}) \), we can express \( P_{out} \) for Case 1 in the high SNR regime as given in (16).

Likewise, the \( P_{out} \) for Case 2 \( (\tau \leq \frac{1}{2}) \) can be obtained and represented by replacing \( \Lambda_{1a} \) and \( \Lambda_{1b} \) with \( \Lambda_{2a} \) and \( \Lambda_{2b} \), respectively, and \( \Theta_{th_1} \) with \( \Theta_{th_2} \), in (16) for Case 1.

To evaluate the asymptotic behavior of \( P_{out} \) for Case 3 in (16), we first reexpress (54) as

\[
P_{out} = \Pr \left[ \frac{\Lambda_{1a} \Lambda_{1b}}{\Lambda_{1a} + \Lambda_{1b}} < \Theta_{th_1}, \frac{\Lambda_{1a} \Lambda_{1b}}{\Lambda_{1a} + \Lambda_{1b}} \right] + \Pr \left[ \frac{\Lambda_{2a} \Lambda_{2b}}{\Lambda_{2a} + \Lambda_{2b}} < \Theta_{th_2}, \frac{\Lambda_{2a} \Lambda_{2b}}{\Lambda_{2a} + \Lambda_{2b}} \right].
\]

(56)

where the first probability term in (56) can be obtained when \( \Lambda_{1b} < \Theta_{th_1} \), which yields \( \Pr \left[ \frac{\Lambda_{1a} \Lambda_{1b}}{\Lambda_{1a} + \Lambda_{1b}} < \Theta_{th_1}, \frac{\Lambda_{1a} \Lambda_{1b}}{\Lambda_{1a} + \Lambda_{1b}} \right] \approx 1 \) and the second probability term in (56) can be expressed when \( \Lambda_{1b} > \Theta_{th_1} \). Likewise, the third and fourth terms in (56) can be evaluated when \( \Lambda_{2a} < \Theta_{th_2} \) and \( \Lambda_{2b} > \Theta_{th_2} \), respectively. Consequently, with the aid of high SNR approximations \( \lim_{\Lambda_{1a} \rightarrow \infty, \Lambda_{1b} \rightarrow \Theta_{th_1}} \Theta_{th_1} = \Theta_{th_2} \) and \( \lim_{\Lambda_{2a} \rightarrow \infty, \Lambda_{2b} \rightarrow \Theta_{th_2}} \Theta_{th_2} = \Theta_{th_2} \), we can further approximate (56) at
high SNR regime as

\[ P_{\text{out}} \overset{\text{asymp}}{\approx} \Pr \left[ \Lambda_1 < \Theta_{\text{th}} \right] \Pr \left[ \Lambda_2 > \delta_{\text{th}} \right] + \Pr \left[ \Lambda_1 < \Theta_{\text{th}} \right] \times \Pr \left[ \Lambda_2 > \delta_{\text{th}} \right] + \Pr \left[ \Lambda_2 > \delta_{\text{th}} \right] \Pr \left[ \Lambda_2 < \delta_{\text{th}} \right] \]  

Simplifying (57) by ignoring higher order infinitesimals, we can get the asymptotic behavior of \( P_{\text{out}} \) for Case 3 in (16).

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